Tuning of a new fuzzy bang–bang relay controller for attitude control system

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Abstract: A new fuzzy bang–bang relay controller (FBBRC) is introduced in this paper. The new controller is inherently optimal due to its bang–bang property. The controller has fuzzy decision-making capability in its inputs and have two fixed levels bang–bang output. Consequently, the tuning of FBBRC is restricted to input parameters only in comparison to standard fuzzy logic controller (FLC) where the output parameters can also be tuned. The stability of new controller stems from well-established bang–bang sliding mode control theory. The work presented here demonstrates that tuning the inputs of the proposed FBBRC is more effective and simpler than tuning all the parameters of standard FLC. The two controllers are tuned online with gradient descent optimisation method and tested for regulator and tracking mode control. Simulation result shows that new controller has faster response time and is capable of controlling the system under adverse initial conditions.

Keywords: self-tuning; fuzzy controller; LOM; largest of maxima; gradient-based optimisation; bang–bang control.


Biographical notes: Farrukh Nagi is working as an Associate Professor in the Department of Mechanical Engineering at Universiti Tenaga Nasional. He graduated from NED Engineering University, Karachi (1982). He worked in Pakistan International Airline for seven years as a Project Engineer on the development of guidance and control systems. He completed his MS from the University of Miami, USA in 1989 and PhD from the University of Nottingham, UK in 1993. From 1994 to 2000, he worked in Pakistan Navy Engineering College – NUST and had undertaken research projects pertaining to submarine acoustic detection. Since 2001, he is working in Universiti Tenaga Nasional and has completed various industrial and research projects. His areas of interest include mechatronics system, fuzzy control and non-linear control systems.
1 Introduction

Four decades after Lofti Zadeh had presented his seminal fuzzy control technique, the selection of fuzzy controller parameter remains in obscurity. Then it is not a surprise that the designers choose the controller parameters heuristically and their expertise in the application area plays an important role in the success of the controller. However, as the demand for high performance controllers grows, the fuzzy controller design process needs to be improved to meet the challenges of the industry. Adaptive tuning and adaptive neuro-fuzzy inference system (ANFIS) are some of the techniques used to eliminate the human interaction in the choice of fuzzy controller parameters. In ANFIS, input/output data of the system are modelled with fuzzy rule-based technique, which is described by the neural network (NN) structure. The NN is trained with back propagation technique to represent the data. Takagi-Sugeno-Kang (TSK) – additive fuzzy mapping model is preferred over ANFIS for simplicity over the non-additive Mamdani models (Yen and Langari, 1999). Adaptive fuzzy controller tuning involves adjustment of existing fuzzy controller’s scale factor (SF) and or membership function (MF). Such tuning is known as self-tuning of the fuzzy logic controller (STFLC) and aims to adapt
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the controller to different operating conditions and to eliminate the disturbance occurring in the process. Fuzzy model limitations, NN-structure complexities and training in ANFIS system make it undesirable for online self-tuning purposes.

PID control is commonly employed in industry but lacks the procedures to select of gain constants. As a result, self-tuning fuzzy PID controller attracts the researcher’s attention. He et al. (1993) present a self-tuning PID control scheme for controlling industrial processes. Ahn et al. (2008) developed a self-tuning PID controller for force control performance in hydraulic load simulator. Yesil et al. (2004) employ the self-tuning PID controller to load frequency control in power engineering. Cammarata and Yliniemi (1999) develop a STFLC for a pilot plant rotary dryer. Similarly, Daugherity et al. (1992) replace the PID controller of simple gas – fired water heater with STFLC. The tuning process requires an optimisation algorithm to adjust the fuzzy parameters, typically the SFs and MFs. Commonly used optimisation algorithms are gradient-based steepest descent, genetic algorithm (GA) and simulated annealing (SA). The most practical online algorithm among the above is gradient method presented by Nomura et al. (1991, 1992) and Li (1993).

Tuning of fuzzy sets has many facets. SF tuning is done externally on the input and output gains of the fuzzy controller. They have critical effect on the response of the fuzzy controller and are easier to tune. Passino and Yurkovich (1998) state that tuning of error input gain has the effect of changing the proportional or loop-gain, resulting in overshoot, while tuning of input – change in error results in altering the derivative gain which affects the transient response of the system, such as settling and rise times. The scaling factor of output MF’s has the effect of increasing the saturation level of the output of the controller. It is not necessary that the fuzzy controller always give satisfactory performance by tuning only gains when subjected to disturbance and non-linearity. Maeda and Murakami (1992) proposed a self-tuning algorithm of the fuzzy logic controller (FLC), which has two functions for adjusting the scaling factors of the FLC, and in improving the control rules (self-organising) of the FLC by evaluating the control response in real time. A more robust tuning method is to tune the MF parameters, such as vertex, shape, spread, order and position (Donato and Barbieri, 1995) or let the optimisation process tune SF and MF at the same time for square error minimisation. Demaya et al. (1995) discuss in detail the effects of tuning SF and MF. They argue that SF has a more profound effect than MF tuning and SF should be used for coarse and MF for the fine-tuning of the response. Nakamura and Kehtarnavaz (1995) used GA and SA techniques and Woodard and Garg (1999) use numerical optimisation to tune the triangular MF parameter – vertex to improve the global performance of the fuzzy controller. Ortega and Giron-Sierra (1998) review the availability of genetic-fuzzy optimisation algorithm for attitude control system.

In most tuning applications, isosceles triangular MFs are used with the TSK or the Mamdani model. Commonly centre of area (COA) or centroid defuzzification is used in fuzzy controller. In this paper, a different defuzzification, largest of maxima (LOM), is used. Arranging the output MF in a certain way gives, bang–bang output from the fuzzy controller. The fuzzy inputs rules and implication remain same as in the conventional fuzzy controller. A major practical advantage of bang–bang controls is that they can be implemented with simple on–off action. Time-optimal control results in bang–bang action (Athans and Palb, 1966; Slotine and Li, 1991) – meaning that over the entire time interval, the control output takes on either its minimum or maximum value to yield minimum-time control of the system. Conventional bang–bang controllers are made from
electromechanical relays that are getting obsolete owing to the fact that their parameters are fixed and act slowly. Solid-state relays are fast acting but are not flexible to control non-linear systems over the entire operating range. The demand for flexible and programmable relays has grown in recent years. Artificial intelligence techniques, such as fuzzy logic, have provided the means to develop flexible fuzzy bang–bang relays. One of the earliest, fuzzy bang–bang controller (FBBC) was developed by Chiang and Jang (1994). Other applications include minimum-time fuzzy satellite attitude controller (Thongchet and Kuntanapreeda, 2001), crane hoisting and lowering operation (Moon et al., 1996), process control valves operation (Usayama and Walden, 1991) and in the reduction of harmonic current pollution (Mazari and Mefri, 2005). The idea of fuzzy relay is not new. Kendal et al. (1991) and Kichert et al. (1978) were first to point out that with mean of maxima (MOM) defuzzification, the fuzzy controller is identical to a multilevel relay. Application of the fuzzy relay in power control was first presented by Panda and Mishra (2000). Hard limiter was used in this work to convert the defuzzified output to two-level control.

Most of the earlier fuzzy bang–bang relay controllers (FBBRCs) were untuned. And the bang–bang controller performance was better than standard fuzzy controller. In this paper, the FBBRC is tuned for minimum-time response and the results are compared on equal terms with tuned standard fuzzy controller. Here, only the input MFs spread and location are tuned. The output MFs of FBBRC are not optimised and bang–bang time interval is optimised by the input MFs for minimisation of objective function. Similarly, the defuzzified output of standard FLC is COA and is not optimised for fair comparison between the two techniques. The rest of this paper is organised as follows. In Section 2, a simple single axis rotary attitude control system is modelled for development of FBBRC and FL controller. In Section 3, the new FBBRC and standard FLC are designed. In Section 4, the stability issues of fuzzy controllers are discussed. In Section 5, the fuzzy tuning optimisation process is presented. In Section 6, simulation results of the two controllers are compared and analysed followed by the conclusion of the work in Section 7.

2 One axis attitude control

A simple one axis attitude control system is described here as an example to develop and demonstrate the FBBRC. This system works on pneumatic and its schematic is shown in Figure 1.

The fuzzy controller has bang–bang action and acts as a regulator to reset the beam to zero reference, \( \theta = 0 \) deg, by firing thrusters \( T_1 \) and \( T_2 \). The equation of motion describing the single axis linear attitude control system is given by

\[
M_a(t) = \dot{\theta}(t)I + \ddot{\theta}(t)C
\]

where \( M_a \) is the moment applied by the thrusters, \( I \) is the moment of inertia of the beam assembly, \( C \) is the coefficient of friction, \( \dot{\theta} \) is the angular rate and \( \ddot{\theta} \) is the angular acceleration.

Equation (1) is graphically modelled in Figure 2, and is simulated to establish and analyse the controllers’ stability and optimality.
The system shown in Figure 1 is modelled with Simulink in Figure 2. The model is used for simulation and passes the system states via in- and out-port blocks to the \textit{m}-file for tuning the fuzzy parameter. The fuzzy tuning is accomplished as shown in Figure 3. Gradient-based steepest descent optimisation algorithm is used for tuning the fuzzy controller.

The specifications of one axis attitude control system are taken from SEDSAT-1 Experimental Satellite (1997) and are reproduced in Table 1.
3 Fuzzy controllers design

Two types of fuzzy controllers are described in this section. Firstly, the new proposed controller, which combines the fuzzy logic with a hard limiter relay in one entity, is presented. This controller is defined as a FBBRC. Secondly, the conventional FLC is presented for comparison. Both controllers use the same input fuzzy sets. However, the output fuzzy sets are different. The FBBRC uses maxima (LOM) defuzzification technique to yield a bang–bang output. The FLC uses the centroid defuzzification technique.

The bang–bang fuzzy relay controller is developed in this section. This controller takes advantage of the LOM defuzzification technique to yield a bang–bang output. For any tuning or non-tuning fuzzy controller, it is necessary to determine the initial ranges of its state input and output variables, which are considered to be a reasonable representation of all the situations that the controller may face and yield to stability and optimality conditions. The following ranges are selected for simulation purposes, \( \theta(t) = [-100, 100] \) deg, \( \dot{\theta}(t) = [-400, 400] \) deg sec\(^{-1}\) and output \( u = [-J, +J] \).

3.1 Description of tuning variables

The inputs and output of the tuning fuzzy controller are shown in Figure 2. The inputs and output parameters, as well as the partitions and spread of the controller MFs are initially selected to match the dynamic response of a pneumatic rotary system. The inputs \( x_i \in X_i \), where \( X_i \) is the universe of discourse of the two inputs, \( i = 1, 2 \). For input variable, \( x_1 = \) ‘error angle’, the tuning universe of discourse, \( X_{1} = [-100, 100] \) deg, which represents the range of perturbation angle about the zero reference. Index \( k \) is assigned to tally the input MFs. For input variable \( x_2 = \) ‘error angle rate’, the tuning universe of discourse is \( X_{2} = [-400, 400] \) deg sec\(^{-1}\). The output universe of discourse \( Y = [-J, +J] \) represents the bang–bang output.

The set \( A_i^k \) is the MF of antecedent part defines as

\[
A_i^k = \begin{bmatrix} A_i^1 \Rightarrow LN, & A_i^2 \Rightarrow SN, & A_i^3 \Rightarrow Zero, & A_i^4 \Rightarrow SP, & A_i^5 \Rightarrow LP \end{bmatrix}
\]

Similar values are selected for input \( x_2 \), \( A_2^k = A_i^k \). The set \( B^k \), which denotes the MF values for output variable \( y \), is defined as:

\[
B^k = \begin{bmatrix} B^1 \rightarrow \text{Nbang}, & B^2 \rightarrow \text{Pbang} \end{bmatrix}
\]
3.2 Tuning fuzzy rules

The fuzzy rules assembled in this work reset the beam angle $\theta = 0$ deg. These rules are based on two input variables, each with five values, thus there are at most 25 possible rules. These rules are described in matrix form in Tables 2 and 3. The shaded diagonal entry in Table 2 is not used. The tuning rules-partitions are heuristically chosen to reset the beam smoothly over the universe of discourse.

The symmetry of the rules matrix is expected as it arises from the symmetry of the system dynamics. The decomposition of the $j$th rule from the FBBRC’s inputs to the output is given by

$$
\mu(y)_{B_j} = \prod_{i=1}^{2} \mu(x_i)_{A_{ij}}
$$

where $j = 1, 2, \ldots, n$ is the index of $n$ matching rules, which are applicable from inferences of inputs. Conventional Fuzzy FBBC uses the standard decomposition technique (Chiang and Jang, 1994; Kichert et al., 1978).

Table 2 Fuzzy rules for FBBRC

<table>
<thead>
<tr>
<th>$\dot{\theta}$</th>
<th>LN</th>
<th>SN</th>
<th>Z</th>
<th>SP</th>
<th>LP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN</td>
<td>+J</td>
<td>+J</td>
<td>+J</td>
<td>+J</td>
<td></td>
</tr>
<tr>
<td>SN</td>
<td>+J</td>
<td>+J</td>
<td>+J</td>
<td></td>
<td>-J</td>
</tr>
<tr>
<td>Z</td>
<td>+J</td>
<td>+J</td>
<td></td>
<td>-J</td>
<td>-J</td>
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<tr>
<td>SP</td>
<td>+J</td>
<td></td>
<td>-J</td>
<td>-J</td>
<td>-J</td>
</tr>
<tr>
<td>LP</td>
<td></td>
<td>-J</td>
<td>-J</td>
<td>-J</td>
<td>-J</td>
</tr>
</tbody>
</table>

Table 3 Fuzzy rules for standard FLC

<table>
<thead>
<tr>
<th>$\dot{\theta}$</th>
<th>LN</th>
<th>SN</th>
<th>Z</th>
<th>SP</th>
<th>LP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN</td>
<td>PL</td>
<td>PL</td>
<td>PL</td>
<td>PS</td>
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<tr>
<td>SN</td>
<td>PL</td>
<td>PL</td>
<td>PS</td>
<td>OFF</td>
<td>NS</td>
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<tr>
<td>Z</td>
<td>PL</td>
<td>PS</td>
<td>OFF</td>
<td>NS</td>
<td>NL</td>
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<td>OFF</td>
<td>NS</td>
<td>NL</td>
<td>NL</td>
</tr>
<tr>
<td>LP</td>
<td>OFF</td>
<td>NS</td>
<td>NL</td>
<td>NL</td>
<td>NL</td>
</tr>
</tbody>
</table>

Note: PL, positive large; PS, positive small and NL, negative large.
3.3 Fuzzy set MFs

The input variables and values assigned to fuzzy set MFs are shown in Figure 4. Triangular shape MFs are used in this work. These MFs are sensitive to small changes that occur in the vicinity of their centres. A small change across the central MF $A_1^3$, located at the origin, can produce abrupt switching of control command $u$ between the +ve and –ve halves of the universe of discourse, resulting in chattering. The overlapping of the central MF $A_1^3$ with the neighboring MFs $A_1^2$ and $A_1^4$ reduce the sensitivity of the bang–bang control action (Passino and Yurkovich, 1998).

Triangular MFs in Figure 4 are based on mathematical characteristics given in Table 4. In Table 4, the $b_i$ and $a_i$ are the tuning parameters for range and central location of MFs, respectively, and shown in Figure 4. Smooth transition between the adjacent MFs is achieved with higher percentage of overlap, which is commonly set to 50%.

The output MFs for standard FLC are shown in Figure 5 and for FBBRC in Figure 6. FBBRC has only two MFs and there is no third central MF at the origin of the output universe of discourse in Figure 6. As a result, there are no diagonal rules in Table 2 as shown by the shaded region. For comparison purposes, the standard FLC (centroid output) and FBBRC use the same input MFs as shown in Figure 4.

\[
y_{COA} = \sum_{i=1}^{n} \frac{\mu_{B_i} w_i}{\sum_{i=1}^{w} \mu_{B_i}}
\]

where $w_i$ is the weight (jet) associated with contributing MF.

Figure 4 Untuned MFs of input $x_1$ = ‘error angle’, $x_2$ = ‘error angle rate’ for both FBBRC and FLC controller
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Table 4  Mathematical characterisation of triangular MFs

<table>
<thead>
<tr>
<th>Linguistic value</th>
<th>Triangular MFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i^{k-1}$</td>
<td>$\mu_{A_i^k}(x) = \begin{cases} \frac{1}{1+\frac{2</td>
</tr>
<tr>
<td>$A_i^2$</td>
<td>$\mu_{A_i^2}(x) = \begin{cases} \frac{1}{1+\frac{2</td>
</tr>
<tr>
<td>$A_i^3$</td>
<td>$\mu_{A_i^3}(x) = \begin{cases} \frac{1}{1+\frac{2</td>
</tr>
<tr>
<td>$A_i^4$</td>
<td>$\mu_{A_i^4}(x) = \begin{cases} \frac{1}{1+\frac{2</td>
</tr>
<tr>
<td>$A_i^5$</td>
<td>$\mu_{A_i^5}(x) = \begin{cases} \frac{1}{1+\frac{2</td>
</tr>
</tbody>
</table>

Figure 5  Output MFs of FLC, min–prod aggregation and centroid defuzzification
LOM uses the union of the fuzzy sets and takes the largest value of the domain with maximal membership degree (Hellendoorn and Thomas, 1993). The output MFs, shown in Figure 6, and the LOM aggregation together formulate the FBBRC. Any perturbation of the beam from the zero reference acts on the output MFs according to the rule matrix in Table 2.

The input MFs of FBBRC are same as described above, which result in same aggregated rules output in Equation (3). The output of FBBRC depends upon the maximum value of degree of MF, $\mu_B$, shown in Figure 6(a).

The defuzzified crisp output $y_{\text{crisp}}$ based on Equation (3) can be evaluated as

$$y_{\text{crisp}} = \arg \sup \{ \mu_B \}$$  \hspace{1cm} (5)

The supremum in Equation (5) is the LOM value and occurs at the extremes of the output universe of discourse $Y = [-J, J]$. The argument $\arg(\sup(\mu))$ returns $y_{\text{crisp}} = [-J, J]$. The bang–bang firing action, $J$ of output MFs $B^*$, is shown in Figure 6(b).

### 4 Bang–bang controller stability

A new controller is often required to guarantee its stability, Giordano (2006) presented GA tuning technique with Lyaponov adaptation law to meet the stability condition. In the case of fuzzy bang–bang controllers (FBBC), the heuristic approach of fuzzy rules result in partitioning of the decision-space (phase plane) into two semi-planes by means of a sliding (switching) line. Similarity between fuzzy bang–bang controller and sliding mode...
controller (SMC) can be used to redefine the diagonal form of FLC in terms of an SMC, with boundary limits, to verify the stability of the proposed bang–bang controller (Chung-Chun and Chia-Chang, 1994; Kulczycki, 2000; Palm et al., 1997). SMC is a robust control method (Slotine and Li, 1991) and its stability is proven with Lyapunov’s direct method. In association with the SMC, the fuzzy bang–bang control stability can be easily established.

The simulated performance of the proposed controller is compared to that of standard FLC with and without hard limiter device, as shown in Figure 2. Any proposed control strategy should be supported by stability analysis for acceptance by the control system community. FBBRC is no exception. As discussed earlier, conventional bang–bang control system has firm stability ground via sliding mode control, which uses Lyapunov like function to satisfy the stability criteria.

4.1 Controller response

The simulation responses of the FBBRC and the standard FLC are shown in Figure 7. Both controllers use the same input MFs, Figure 4, and initial conditions. However, the output MFs are different. The FLC is simulated with and without the hard limiting function. The result shows that the overshoot and settling time is less for the FBBRC.

The fuzzy rules described in Table 2 can be systematically constructed on the basis of sliding mode control and hitting condition described by Equation (A13) in the Appendix. Appendix provides detailed derivation of the control law and stability condition of the SMC. The state trajectory of SMC controller chatters along the sliding line to zero in Figure 8, while FBBRC follows smooth curve path to join the sliding line just before zero. Consequently, the FBBRC avoid sliding mode chattering and reaches zero in shorter time as shown in Figure 7. FLC does not follow sliding mode at all, as it does not have bang–bang action.

Figure 7 Controllers response comparison from initial conditions [20 deg, 3.142 deg sec\(^{-1}\)]

Note: FBBRC has the lowest overshoot and settling time.
4.2 Fuzzy SMC

The rules in Table 2 can be deduced from Equation (A8). Multiplying it with \( s \) yields

\[
\dot{s} = f(\theta; t)s + b(\theta; t)us + \lambda \dot{\theta}s
\]  

(6)

For \( b > 0 \), if \( s < 0 \), then increasing \( u \) will result in decreasing \( s \); and that if \( s > 0 \), then decreasing \( u \) will result in decreasing \( s \). The control value \( u \) should be selected so that \( s < 0 \) for \( 0 < s > 0 \). The slope of sliding line is represented by \( \lambda \).

Considering \( s \) as \( \theta \) and \( \dot{s} \) as \( \dot{\theta} \), then for \( J = 1 \), \( u = [-1, +1] \), the fuzzy rules in Table 2 and the MFs shown in Figure 6(a) agree with the sliding mode condition.

4.3 Tuning conditions of controllers

The fuzzy set described above satisfies the following conditions:

1. Membership function range variable \( b \), Figure 4, act upon the bordering input MFs \( A_1^i \) and \( A_5^i \), and tunes the SF of the inputs, Figure 4. This has an effect on the proportional gain, which changes sharply in the beginning of the optimisation process and also optimises the overlaps between the MFs.

2. The input’s central MF \( A_3^i \) is fixed at zero to keep the symmetry in the control as required by the dynamics of the system.

3. The input MFs \( A_2^i \) and \( A_4^i \) in Figure 4, are allowed to change their central value \( a_i \), and has an effect of fine tuning the response in the vicinity of the desired response.
4 During the optimisation/tuning process Bezdek’s repartition is satisfied, i.e. maximum (1) of a MF corresponds to minimum (0) of the adjacent MF.

5 The order of MFs NL, NS, zero, PS and PL is always respected according to Bezdek’s distribution (Passino and Yurkovich, 1998), i.e. the modal value of any MF never crosses the modal value of another MF.

5 Fuzzy tuning optimisation

The optimisation process uses gradient-based steepest descent method (Nomura and Hayashi, 1992). This method gets the vector $Z$ which minimises a objective function $E(Z)$, where $Z = [z_1 = b_{1}^{k+1}, z_2 = b_{1}^{k+5}, z_3 = a_{k}^{k+2}, z_4 = a_{k}^{k+4}]$. By optimisation iterations, the variation of $Z$ which decrease the objective function $E(Z)$ is expressed by

$$\left[ \frac{\partial E}{\partial z_1}, \frac{\partial E}{\partial z_2}, \frac{\partial E}{\partial z_3}, \frac{\partial E}{\partial z_4} \right]$$

Therefore, tuning of each parameter is defined as follows:

$$z_i(t+1) = z_i - K \frac{\partial E(Z)}{\partial z_i}$$

(7)

where $t$ is the number of iteration required to reach a error limit and $K$ is a constant. When $Z$ is tuned according to Equation (6), the objective function $E(Z)$ converges to a local minimum. In this paper, the objective $E(Z)$ is defined as the inference error between the desired output $y_r$ and the actual output, $y = y_{\text{COA}}$ and $y = y_{\text{cisp}}$, from Equation (3) to (6), respectively.

$$E = \frac{1}{2} (y - y_r)^2$$

(8)

According to Equation (7), the update of parameters is accomplished as

$$a_{i,j}(t+1) = a_{i,j} - K_a \frac{\partial E}{\partial a_{i,j}}$$

$$b_{i,j}(t+1) = b_{i,j} - K_b \frac{\partial E}{\partial b_{i,j}}$$

$$w_i(t+1) = w_i - K_w \frac{\partial E}{\partial w_i}$$

(9)

$K_a$, $K_b$ and $K_w$ are constants to control the rate of convergence of the optimisation process and $(t + 1)$ is the update value after each iteration. Note index $j$ added to the tuning parameters $a_{i,j}$ and $b_{i,j}$ to account for only those rules, which are contributing, to the controller output.

The gradient of the objective function

$$\left[ \frac{\partial E}{\partial a_{i,j}}, \frac{\partial E}{\partial b_{i,j}}, \frac{\partial E}{\partial w_i} \right]$$
in (9) can be derived from Table 4, Equations (3) and (8) with chain rule as

\[
\frac{\partial E}{\partial a_{i,j}} = \frac{\partial E}{\partial y} \times \frac{\partial y}{\partial \mu_i} \times \frac{\partial \mu_i}{\partial A_{i,j}} \times \frac{\partial A_{i,j}}{\partial a_{i,j}} \tag{10}
\]

where

\[
\frac{\partial y}{\partial y} = \left( y - y_r \right)
\]

\[
\frac{\partial y}{\partial \mu_i} = \frac{w_i}{\sum \mu_i} - \frac{w_i w_i w_i}{\left( \sum \mu_i \right)^2} = \frac{w_i - y}{\sum \mu_i}
\]

\[
\frac{\partial \mu_i}{\partial A_{i,j}} = \frac{\mu_i}{A_{i,j}}
\]

\[
\frac{\partial A_{i,j}}{\partial a_{i,j}} = 2 \sgn \left( x_j - a_{i,j} \right)
\]

Then from Equation (10)

\[
\frac{\partial E}{\partial a_{i,j}} = \left( y - y_r \right) \frac{w_i - y}{\sum \mu_i} \frac{\mu_i}{A_{i,j}} \frac{2}{b_{i,j}} \sgn \left( x_j - a_{i,j} \right) \tag{12}
\]

and

\[
\frac{\partial E}{\partial b_{i,j}} = \frac{\partial E}{\partial y} \times \frac{\partial y}{\partial \mu_i} \times \frac{\partial \mu_i}{\partial A_{i,j}} \times \frac{\partial A_{i,j}}{\partial b_{i,j}} \tag{13}
\]

where

\[
\frac{\partial A_{i,j}}{\partial b_{i,j}} = - \left[ \frac{b_{i,j} - 2x_j - a_{i,j}}{b_{i,j}} \right] + b_{i,j}^{-1}
\]

\[
= \left[ \frac{1 - A_{i,j}(x_j)}{b_{i,j}} \right] \tag{14}
\]

Then from Equations (11) and (14)

\[
\frac{\partial E}{\partial b_{i,j}} = \left( y - y_r \right) \frac{w_i - y}{\sum \mu_i} \frac{\mu_i}{A_{i,j}} \left[ \frac{1 - A_{i,j}(x_j)}{b_{i,j}} \right] \tag{15}
\]
Further

\[ \frac{\partial E}{\partial w_i} = (y - y_r) \sum \mu_i \] (16)

Putting Equations (12), (15) and (16) back in Equation (9) give the results of the most recent iteration.

\[ a_{i,j}(t+1) = a_{i,j} - K_a y - y_r \left(y - y_r\right) \frac{w_j - y}{\sum \mu_i A_{i,j} b_{i,j}} \cdot \frac{2}{\mu_i} \cdot \text{sgn}\left(x_j - a_{i,j}\right) \]

\[ b_{i,j}(t+1) = b_{i,j} - K_b \left(y - y_r\right) \frac{w_j - y}{\sum \mu_i A_{i,j} b_{i,j}} \cdot \left[1 - A_{i,j}\left(x_j\right)\right] \] (17)

\[ w_j(t+1) = w_j - K_w \left(y - y_r\right) \frac{\mu_i}{\sum \mu_i} \]

The tuning parameters \( b_{i,j} \) and \( a_{i,j} \) minimise the objective function \( E \). The above equations are iteratively solved until the error \( e \) reaches a specified threshold level.

6 Simulation response

Attitude linear system in Equation (1) is stable per se and here tuning of proposed FBBRC and conventional FLC controllers is compared. The model in Figure 2 is simulated for this purpose. Steepest descent, gradient-based optimisation algorithm described in previous section is used for tuning the MF parameters \( a_{i,j} \) and \( b_{i,j} \). The controllers’ performances are evaluated and compared in regulator and tracking mode.

6.1 Regulator mode

In regulator mode, initial angle and angular rate are input to test the two controllers. It was found that the relative signs of the inputs distinguish the results between the two controllers.

In Figure 9, the system is controlled without tuning of MFs parameters, which are shown in Figures 4–6. The initial angle and angular velocity in same direction imply that the beam is moving away from the zero reference and the controller has to arrest the motion before turning it around (change direction) towards zero reference, as shown in Figure 9 – left.

This requires considerable effort +\( u \) (ON-time) from the controller, which FBBRC delivers to the plant efficiently in comparison to FLC. In Figure 9 – right, the initial angle and angular velocity are in opposite direction, meaning that the velocity direction is towards the zero reference, so less resetting force, −\( u \) (ON-time) is required by the
controller and both the FLC and FBBRC are in close proximity to each other. For untuned condition, the FBBRC has shorter resetting time and tuning reduce the overall resetting time of both the controller with better performance of FBBRC in case of opposite direction of velocity and angle.

### 6.2 Tracking mode

The tracking capabilities of the FBBRC in comparison to FLC are shown in Figure 10. The two controllers perform close to each other. The effect of opposite sign of velocity and angle in Steps 1–3 shows that FBBRC has less overshoot and better settling time as before. Further, the maximum bang–bang effort enables the FBBRC to slew large tracking angle and velocities without violating the Bezdek’s repartition criteria.

**Figure 9** Comparison between the proposed FBBRC and the conventional FLC: *left*: initial angle and velocity in same direction; *right*: initial angle and velocity in opposite direction; *top*: before tuning; *centre*: after tuning both the controllers and *bottom*: the control effort required by both the controllers.
7 Conclusion

In this paper, tuning of a new FBBRC is presented. The proposed scheme has stability support of sliding mode control due to its proximity with non-linear bang–bang control theory. The new controller is simple in configuration with two-level output, similar to bang–bang relay controls and yet has a fuzzy decision-making capability on its inputs side. The front-end inputs are similar to standard fuzzy controllers based upon Mamdani implication but have a LOM defuzzification output. The new controller performs better with or without tuning in comparison to the FLC. The simulation results confirm the dynamic control capabilities of the FBBRC are superior to FLC under adverse conditions.

References


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Appendix

**Sliding mode control (SMC)**

A general second-order non-linear single-input-single-output (SISO) control system could be described (Donato and Barbieri, 1995) as

\[
\dot{\theta}(t) = f(\theta(t), t) + b(\theta(t))u(t)
\]

(A1)

where \( \theta(t) \) is the output of interest, \( u(t) \) is the scalar input and \( \theta = [\Phi, \dot{\Phi}]^T \) is the state vector. In general, \( f(\theta(t), t) \) is not precisely known, but upper bounded by a known continuous function of \( \theta \). Similarly, \( b(\theta(t), t) \) is not known, but is of known sign and is bounded by a known continuous function of \( x \) as

\[
|f - \hat{f}| \leq F(\theta(t))
\]

\[
\frac{1}{\beta(\theta(t))} \leq \frac{\hat{b}}{b} \leq \beta(\theta(t))
\]

(A2)

where \( \hat{f} \) and \( \hat{b} \) are the nominal values of \( f \) and \( b \), respectively, without the function argument for brevity purpose.

Comparing Equations (1) and (A1), the system becomes:

\[
\dot{\theta}(t) = -\frac{1}{I} \theta(t)
\]

\[
b(\theta(t), t) u(t) = \frac{M}{I} u(t) \quad \therefore b(\theta(t), t) = \frac{M}{I}
\]

(A3)

where \( u(t) \) is a unit step input.

The control problem is to get the state \( \theta \) to track \( \theta_d = [\theta_d, \dot{\theta}_d]^T \) in minimum time and in the presence of imprecise friction. The initial \( \theta_d \) should be the following in view of finite control \( u \)

\[
\theta_d(0) = \theta(0)
\]

(A4)

The tracking error between the actual and desired state would be

\[
e = \theta - \theta_d \pm [e_e \dot{e}]^T
\]

(A5)

A sliding – switching line \( s(\theta, t) \) in the second-order state space \( \mathbb{R}^2 \) is defined such that \( e \) follows the line \( s(\theta, t) = 0 \). The sliding line \( s(\theta, t) \) is determined by

\[
s(\theta, t) = \left( \frac{d}{dt} + \lambda \right)^{n-1} e
\]

(A6)

Equation (A6) can be expanded with binomial expansion and \( \lambda \) is positive constant. For \( n = 2 \)

\[
s = \dot{e} + \lambda e \quad \therefore \dot{e} = \dot{\theta} \quad \& \quad e = \theta
\]

(A7)
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Then from Equation (A)
\[ \dot{s} = f(\theta, t) + b(\theta, t)u + \lambda \dot{\varepsilon} \] (A8)

SMC control law

Let \( u_{eq} \) be the equivalent control law that will keep the states on the sliding trajectory, computed by \( \dot{s} = 0 \) for \( u = u_{eq} \) then from Equations (A4), (A5) and (A7)
\[ s = \dot{\theta} + \lambda \theta \]
\[ \dot{s} = \dot{\theta} + \lambda \dot{\theta} \] (A9)

Then from Equation (A8) with uncertainties
\[ \dot{s} \left|_{u = u_{eq}} \right. = \dot{f}(\theta, t) + \dot{b}(\theta, t)u_{eq} + \lambda \dot{\varepsilon} = 0 \]

Solving the above equation
\[ u_{eq} = \hat{b}^{-1}\hat{u} \] (A10)

where
\[ \hat{u} = \left[ -\dot{f}(\theta, t) - \lambda \dot{\varepsilon} \right] \] (A11)

or
\[ \lambda \dot{\varepsilon} = -\dot{f}(\theta, t) - \hat{u} \]

is the nominal control input in presence of uncertainties?

SMC – reaching condition

The control input \( u \) to get the state \( \theta \) to track \( \theta_d \) is then made to satisfy the Lyapunov-like function \( V = (1/2)s^2 \), if there exist \( \eta > 0 \) and by the following sliding condition (Donato and Barbieri, 1995):
\[ \frac{1}{2} \frac{d}{dt} s^2(\theta, t) \leq -\eta |s| \] (A12)

Which is reduced to the so-called sliding mode reaching condition for Equation (1)
\[ \dot{s} \times \text{sgn}(s) \leq -\eta |s| \quad \eta > 0 \] (A13)

The control law that satisfies the sliding mode reaching conditions Equation (A13) can be obtained as
\[ u = u_{eq} + u_s \] (A14)

where
\[ u_s = -K \text{sgn}(s) \] (A15)
and
\[ \text{sgn}(s) = \begin{cases} +1, & \text{if } s > 0 \\ -1, & \text{if } s < 0 \end{cases} \]

Substituting Equations (A1) and (A8) in Equation (A13)
\[ ss = s(f + bu + \lambda \dot{e}) \leq -\eta|s| \]

*Note*: Here we have dropped the function argument for brevity purpose. Then equivalently we can write:
\[ ss = \text{sgn}(s)(f + \lambda \dot{e}) + bu \text{sgn}(s) \leq -\eta|s| \]  
(A16)

Substituting Equations (A14) and (A15) into Equation (A16)
\[ ss = \text{sgn}(s)(f + \lambda \dot{e}) + b \left[ u_{eq} + \dot{b}^{-1} K \text{sgn}(s) \right] \text{sgn}(s) \leq -\eta|s| \]

Substituting from Equations (A10) and (A11) in above, we get
\[ ss = \text{sgn}(s) \left[ f + \left( -\dot{f} - \dot{u} \right) \right] + b \left[ \dot{b}^{-1} \dot{u} + \dot{b}^{-1} K \text{sgn}(s) \right] \text{sgn}(s) \leq -\eta|s| \]

simplifying we get
\[ \text{sgn}(s) \left( f - \dot{f} \right) + \left[ \frac{b}{\dot{b}} - 1 \right] \dot{u} \text{sgn}(s) - \frac{b}{\dot{b}} K \leq -\eta|s| \]  
(A17)

Then for upper bounds from Equation (A1) need
\[ K \geq \beta(F + \eta) + (\beta - 1) \dot{u} \]  
(A18)

to satisfies the reaching or hitting condition.